Summary of the first part of Unit 1
"From lesson 1 to lesson 5"

- The cube root of the number "a" is the number whose cube equals a
  For example : $\sqrt[3]{64} = 4$, $\sqrt[3]{-64} = -4$
- The cube root of the positive number is positive, and the cube root of the negative number is negative.
- $\sqrt[3]{a^3} = a$ For example : $\sqrt[3]{(-5)^3} = -5$
  , $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$
  For example : $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$
- If "a" is a perfect cube number, then the equation : $x^3 = a$ has a unique solution in $\mathbb{R}$
  , which is $\sqrt[3]{a}$
- Each irrational number lies between two rational numbers and can be represented by a point on the number line.
- The set of rational numbers $\mathbb{Q}$ and the set of irrational numbers $\grave{\mathbb{Q}}$ are disjoint sets.
  i.e. $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$
- $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$, $\mathbb{R} - \mathbb{Q} = \grave{\mathbb{Q}}$, $\mathbb{R} - \grave{\mathbb{Q}} = \mathbb{Q}$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_- =]-\infty , \infty[$, $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
  , $\mathbb{R}_+ =]0 , \infty[$, $\mathbb{R}_- =]-\infty , 0[$
  , the set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$
  , the set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$
  , $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$
- It is possible to carry out the operations of intersection , union, difference and complement on the intervals.
- The set of real numbers is closed under addition, subtraction and multiplication operations and is not closed under division operation.
- Each of addition and multiplication operation in $\mathbb{R}$ is commutative and associative, but each of subtraction and division operation in $\mathbb{R}$ is not commutative and associative.
- Zero is the additive neutral in $\mathbb{R}$, and one is the multiplicative neutral in $\mathbb{R}$
- For every real number "a" , there is an additive inverse which is the real number "– a", and for every real number "a" where $a \neq 0$, there is a multiplicative inverse which is the real number $\frac{1}{a}$
- The multiplication in the set of real numbers is distributed on the addition and the subtraction from right and from left.

Exams on the first part of unit one
from lesson (1) to lesson (5)

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 $[-2 , 5] - \{-2 , 5\} = \dots\dots$

(a) $\{-2 , 5\}$ (b) $[-2 , 5[$ (c) $]-2 , 5[$ (d) $]-2 , 5]$

2 $\mathbb{Q} \cap \grave{\mathbb{Q}} = \dots\dots$

(a) $\mathbb{Z}$ (b) $\mathbb{R}$ (c) $\grave{\mathbb{Q}}$ (d) $\varnothing$

3 $\sqrt{x^4} = \sqrt[3]{\dots\dots}$

(a) x^6 (b) x^4 (c) x^2 (d) x

4 The irrational number included between 3 and 4 is $\dots\dots$

(a) $\sqrt{7}$ (b) $\sqrt{10}$ (c) $\sqrt[3]{12}$ (d) $3\frac{1}{4}$

5 The multiplicative inverse of the number $\sqrt{3}$ is $\dots\dots$

(a) $\frac{3}{\sqrt{3}}$ (b) -3 (c) 3 (d) $\frac{\sqrt{3}}{3}$

6 If $\frac{x}{4} = \frac{16}{x^2}$, then $x = \dots\dots$

(a) 2 (b) 4 (c) 8 (d) 16

2 Complete the following :

1 $\sqrt[3]{4 + \dots\dots} = 3$

2 The square whose side length is $\sqrt{5}$ cm., its area is $\dots\dots$ cm^2.

3 $\mathbb{R} - \mathbb{Q} = \dots\dots$

4 $[-2 , 7] \cap]-2 , 7[= \dots\dots$

5 The additive inverse of the number $5 - \sqrt{3}$ is $\dots\dots$

3 [a] If $X = [-2 , 3]$ and $Y = [1 , 5[$, find using the number line each of :

1 $X \cap Y$ 2 $X \cup Y$ 3 $Y - X$

[b] Find the solution set in $\mathbb{R}$ of the equation : $(x^2 - 4)(x^3 - 7) = 0$

Unit 1

4 [a] Prove that $\sqrt{12}$ is included between 3.4 and 3.5

[b] A square of side length 5 cm. , find its diagonal length.

5 [a] Determine the point which represents the number $\sqrt{5}$ on the number line.

[b] **Find the result of each of the following operations :**

1 $(\sqrt{3}+1)(\sqrt{3}-1)$ 2 $(\sqrt{7}+2)(\sqrt{7}-1)$

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $\mathbb{R} =$ ....

(a) $\mathbb{R}_+ \cup \mathbb{R}_-$ (b) $\mathbb{R}_+ \cap \mathbb{R}_-$ (c) $\mathbb{Q} \cup \grave{\mathbb{Q}}$ (d) $\mathbb{R}^*$

2 The irrational number from the following numbers is ....

(a) $\sqrt{\frac{4}{25}}$ (b) $\sqrt[3]{1}$ (c) $\sqrt{\frac{27}{8}}$ (d) $\sqrt[3]{\frac{1}{64}}$

3 If $-\sqrt{4} = \sqrt[3]{x}$, then $x =$ ....

(a) 8 (b) – 8 (c) 4 (d) 16

4 $\sqrt{4} - \sqrt[3]{-8} =$ ....

(a) –2 (b) 4 (c) – 4 (d) 8

5 $\{2 , 5 , 7\} -]2 , 7] =$ ....

(a) $\{2\}$ (b) $\{2 , 5\}$ (c) $]2 , 5[$ (d) $[2 , 5]$

6 $\sqrt{4}$ .... $]-2 , \infty[$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

2 **Complete the following :**

1 The additive inverse of the number $\sqrt{7} - \sqrt{2}$ is ....

2 $\mathbb{R}_+ \cup \mathbb{R}_- =$ ....

3 $[-4 , 6[- \mathbb{R}_+ =$ ....

4 The sum of the real numbers in the interval $[-3 , 3[$ is ....

5 The solution set of the equation $x^2 + 25 = 0$ in $\mathbb{R}$ is ....

3 **[a] Find the result of the following in the simplest form :**

$2\sqrt{7}-5\sqrt{2}+\sqrt{7}+5\sqrt{2}$

[b] If $X =]-\infty , 1[$ and $Y = [-2 , 4[$ **, using the number line find in the form of an interval each of the following :**

1 $X \cup Y$ 2 $X \cap Y$ 3 $\grave{X}$

4 **[a] Find in $\grave{\mathbb{Q}}$ the solution set of each of the following equations :**

1 $\frac{1}{2} x^2 - 3 = 7$ 2 $125 x^3 - 3 = 5$

[b] Prove that $\sqrt[3]{17}$ is included between 2.57 and 2.58

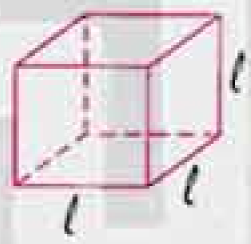
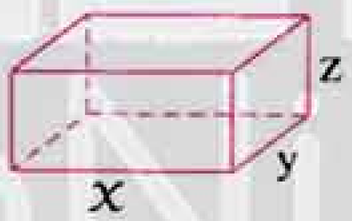
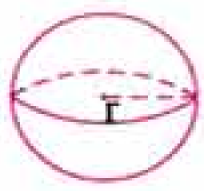
5 **[a] Simplify to the simplest form :**

1 $(2\sqrt{3}-5)^2$ 2 $\sqrt{5}(\sqrt{5}+2)$

[b] Write four irrational numbers included between 11 and 12

Summary of the second part of Unit 1
"From lesson 6 to lesson 10"

- If a and b are two non-negative real numbers , then :
  - $\sqrt{a} \times \sqrt{b} = \sqrt{a\,b}$
  - $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ (where $b \neq 0$)
- If a and b are two real numbers , then :
  - $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{a\,b}$
  - $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\dfrac{a}{b}}$ (where $b \neq 0$)
- If a and b are two positive rational numbers , then each of the two numbers $\left(\sqrt{a} + \sqrt{b}\right)$ and $\left(\sqrt{a} - \sqrt{b}\right)$ is conjugate to the other one and :
  - Their sum $= 2\sqrt{a}$
  - Their product $= a - b$
- If we have a real number whose denominator is written in the form $\left(\sqrt{a} + \sqrt{b}\right)$ or $\left(\sqrt{a} - \sqrt{b}\right)$, we put it in the simplest form by multiplying both the numerator and the denominator by the conjugate of the denominator.
- The following table summarizes the rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4\,\ell^2$	$6\,\ell^2$	ℓ^3
The cuboid		$2\,(x + y) \times z$	$2\,(xy + yz + zx)$	$x\,y\,z$
The cylinder		$2\,\pi\,r\,h$	$2\,\pi\,r\,h + 2\,\pi\,r^2 = 2\,\pi\,r\,(h + r)$	$\pi\,r^2\,h$
The sphere		–	$4\,\pi\,r^2$	$\frac{4}{3}\,\pi\,r^3$

Remember that : the circumference of the circle $= 2\,\pi\,r$, the area of the circle $= \pi\,r^2$

- Solving the equation or the inequality is finding the values of the unknown which satisfy this equation or inequality.
- The solution set of the inequality of the first degree in one variable in $\mathbb{R}$ is written in the form of an interval.

Exams on the second part of unit one
from lesson (6) to lesson (10)

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The volume of the sphere of diameter length 3 cm. equals .... cm^3.
   (a) 4.5π (b) 36π (c) 288π (d) 4.5
2. If $x > 3$, then $-x$ ....
   (a) < 3 (b) > -3 (c) < -3 (d) $< \frac{-1}{3}$
3. $\sqrt{20} - \sqrt{5} =$ ....
   (a) $\sqrt{15}$ (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 15
4. A cube, its volume is $125\ cm^3$, then its total area equals .... cm^2.
   (a) 30 (b) 25 (c) 100 (d) 150
5. If $x = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{7} - \sqrt{3}$, then $xy =$ ....
   (a) 4 (b) 10 (c) 40 (d) 58
6. $\frac{\sqrt[3]{24}}{\sqrt[3]{3}} =$ ....
   (a) 8 (b) 3 (c) 2 (d) $\sqrt[3]{2}$

2 **Complete the following :**

1. The multiplicative inverse of the number $(\sqrt{3} - \sqrt{2})$ in the simplest form is ....
2. $\sqrt{2} \times \sqrt{12} = 2 \times$ ....
3. $\sqrt[3]{54} - \sqrt[3]{2} =$ .... (in the simplest form)
4. A right circular cylinder, its volume is $500\pi\ cm^3$ and the diameter length of its base is 10 cm., then its height is ....
5. If $1 - x > 5$, then x ....

3 **[a] Find in the simplest form the value of the expression :**

$$\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$$

[b] A sphere, its volume is $36\pi\ cm^3$. Calculate its area.

Unit 1

4 **[a] Find in $\mathbb{R}$ the solution set of the inequality :**

$-3 < 2x + 1 < 7$, then represent it on the number line.

[b] A right circular cylinder , its height equals the radius length of its base and its volume is 27π cm^3. Find the radius length of its base.

5 **[a] Simplify to the simplest form :** $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

[b] If $x = \sqrt{5} - \sqrt{2}$ and $y = \frac{3}{\sqrt{5} - \sqrt{2}}$

Prove that : x and y are conjugate , then find : $x^2 + 2xy + y^2$

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $\sqrt{3\frac{3}{8}} = \frac{3}{2}\sqrt{\frac{\cdots\cdots}{\cdots\cdots}}$

   (a) $\frac{3}{8}$ (b) $\frac{3}{2}$ (c) $\frac{27}{8}$ (d) $\frac{729}{64}$

2. The number $(1 - \sqrt{3})(1 + \sqrt{3})$ is .... number.

   (a) a natural (b) a rational (c) an irrational (d) a prime

3. $\sqrt{3} + \sqrt{3} = \cdots\cdots$

   (a) 3 (b) $\sqrt{6}$ (c) $2\sqrt{6}$ (d) $2\sqrt{3}$

4. A sphere , its volume is $\frac{4}{3}\pi$ cm^3 , then its diameter length is .... cm.

   (a) 0 (b) 1 (c) 2 (d) $\frac{4}{3}$

5. A cube , its volume is $2\sqrt{2}$ cm^3 , then its edge length equals .... cm.

   (a) $\sqrt{2}$ (b) 2 (c) 8 (d) 4

6. $\sqrt[3]{2} \times \sqrt[3]{2} = \cdots\cdots$

   (a) 2 (b) 4 (c) $\sqrt[3]{4}$ (d) $\sqrt{2}$

2 **Complete the following :**

1. If $x = \frac{1}{\sqrt{8} - \sqrt{5}}$ and $xy = 1$, then y = ...

2 The solution set of the inequality : $4 > -2x$ in $\mathbb{R}$ is ....

3 $\frac{\sqrt[3]{32}}{\sqrt[3]{4}} =$ ....

4 $\left(\sqrt{8}+\sqrt{2}\right)^2 =$ ....

5 A right circular cylinder , its volume is 90π cm^3 , and its height is 10 cm. , then the radius length of its base equals .... cm.

3 **[a] Prove that :** $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=0$

[b] If $a=\frac{4}{\sqrt{7}-\sqrt{3}}$ and $b=\sqrt{7}-\sqrt{3}$

, find in the simplest form : $\frac{a-b}{a\,b}$

4 **[a] Simplify :**

$2\sqrt{5}+9\sqrt{\frac{1}{3}}-\sqrt{27}-5\sqrt{\frac{1}{5}}$

[b] A right circular cylinder with volume 36π cm^3 and its height is 4 cm. , and the radius length of its base equals the edge length of a cube. Find the total area of the cube.

5 **[a]** A right circular cylinder , its volume is 231 cm^3 , and its height is 6 cm.

Calculate its lateral area $\left(\pi=\frac{22}{7}\right)$

[b] Find in $\mathbb{R}$ the solution set of the inequality :

$5x-3<2x+9$ and represent it on the number line.

Summary of Unit 2

- The linear relation is a relation of the first degree between two variables x and y , it is in the form : $a x + b y = c$ where a , b and c are real numbers , a and b are not both equal to zero , and there is an infinite number of ordered pairs which satisfy this relation , and it is represented graphically by a straight line.
- To graph a linear relation , you need to graph at least two ordered pairs satisfying this relation , you can add a third ordered pair to check that the three points lie on the same straight line which is the graphic representation of the relation.
- The relation : $y = 0$ is represented by x-axis.
- The relation : $x = 0$ is represented by y-axis.
- The linear relation $a x + b y = 0$ is represented graphically by a straight line passing through the origin point.
- The slope of the straight line $= \frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

  i.e. $S = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$
- The slope of the straight line parallel to x-axis equals zero
- The slope of the straight line parallel to y-axis is undefined.

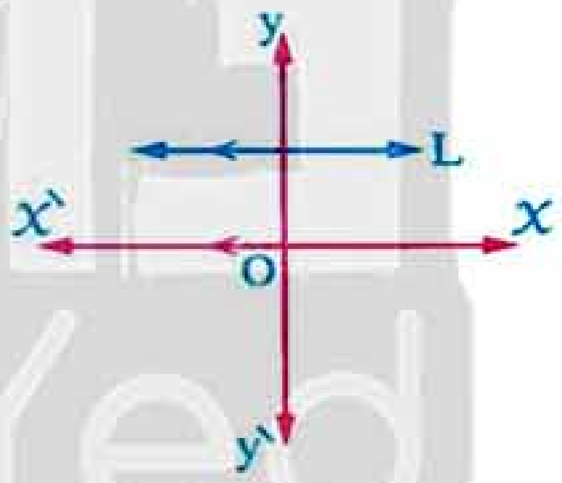
Exams on Unit Two

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. Which of the following ordered pairs satisfies the relation : $2x + y = 5$?
   (a) $(-3, -1)$ (b) $(3, 1)$ (c) $(1, 3)$ (d) $(2, 2)$
2. If $(2k, k)$ satisfies the relation : $y + 2x = 5$, then $k = \cdots\cdots$
   (a) 1 (b) 2 (c) 3 (d) 4
3. The slope of the straight line passing through the two points $(2, 3)$, $(-5, 3)$ is $\cdots\cdots$
   (a) 2 (b) 1 (c) 0 (d) undefined.
4. The relation : $x - 3 = 0$ is represented by a straight line of slope $\cdots\cdots$
   (a) 0 (b) undefined (c) 5 (d) -5
5. **In the opposite figure :**
   The slope of the straight line L is $\cdots\cdots$
   (a) positive. (b) negative.
   (c) 0 (d) undefined.



6. $(3, 1)$ does not satisfy the relation $\cdots\cdots$
   (a) $y + x = 4$ (b) $2x - y = 5$
   (c) $3y + x = 4$ (d) $4y + 2x = 10$

2 **Complete the following :**

1. The relation : $3x + 4y = 12$ is represented by a straight line intersecting the x-axis at the point $\cdots\cdots$
2. If the slope of the straight line passing through the two points $(3, y)$, $(5, -2)$ is -3 , then $y = \cdots\cdots$
3. If $(-1, 5)$ satisfies the relation : $3x + ky = 7$, then $k = \cdots\cdots$
4. The slope of the straight line that is parallel to the y-axis is $\cdots\cdots$
5. If the straight line : $ax + by + c = 0$ passes through the origin point , then $c = \cdots\cdots$

3 **[a]** Represent graphically the relation : $2x + y = 4$

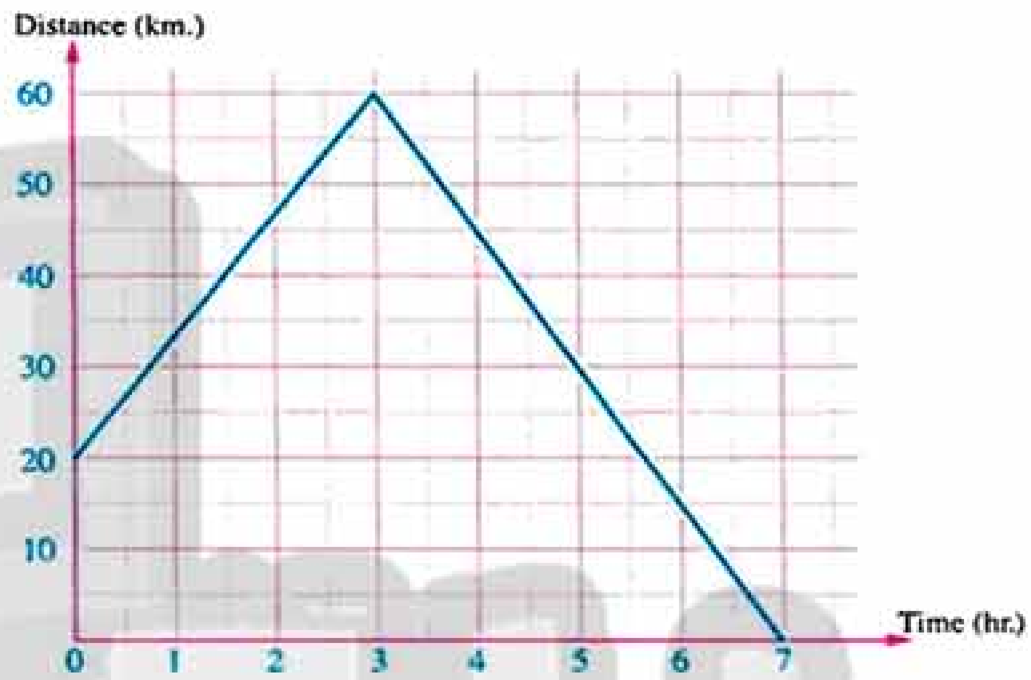
[b] Prove that the points A $(4, 3)$, B $(1, 1)$ and C $(-5, -3)$ are collinear.

Unit 2

4 **[a]** Represent graphically the straight line that represents the relation : $2y - 3x = -6$ and if the straight line intersects the x-axis at the point A and intersects the y-axis at the point B , find the area of Δ OAB where O is the origin point.

[b] Find the value of y such that the straight line passing through the two points $(4, -1)$, $(-2, 2y)$ is perpendicular to the y-axis.

5 The opposite figure represents the movement of a bicycle from a fixed point.



Find :

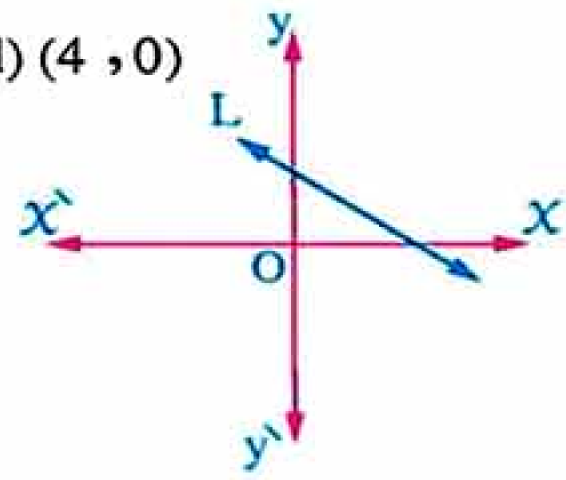
1. The velocity of the bicycle during the first three hours.
2. The velocity of the bicycle during the next four hours.
3. The total distance.

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The ordered pair which does not satisfy the relation : $y = x + 1$ is ....
   (a) $(0, 1)$ (b) $(2, 3)$ (c) $(1, 2)$ (d) $(2, 5)$
2. If $(5, 2m)$ satisfies the relation : $y = 3x - 1$, then $m =$ ....
   (a) 2 (b) 7 (c) 10 (d) 14
3. If the slope of the straight line representing the relation $x + my = 5$ is undefined , then $m =$ ....
   (a) 1 (b) -1 (c) 5 (d) zero.
4. The relation : $2x + 3y = 12$ is represented by a straight line intersecting the y-axis at the point ....
   (a) $(6, 0)$ (b) $(0, 6)$ (c) $(0, 4)$ (d) $(4, 0)$
5. **In the opposite figure :**
   The slope of the straight line L is ....
   (a) positive. (b) negative.
   (c) zero. (d) undefined.



6 The slope of the straight line $\overleftrightarrow{yy}$ is ·········

(a) zero. (b) undefined. (c) 1 (d) – 1

2 Complete the following :

1 The slope of the straight line parallel to X-axis is ·········

2 If (2 , –1) satisfies the relation : $2X + 3y + c = 0$, then c = ·········

3 The straight line which represents the relation :
$y = 2X + 5$ intersects X-axis at the point ·········

4 The relation : $X - 5 = 0$ is represented by a straight line whose slope is ·········

5 If the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$, then A , B and C are ·········

3

[a] Represent graphically the relation : $y - 2X + 1 = 0$

[b] If the straight line which represents the relation : $X - 2y = a$ intersects y-axis at the point (b , 3) , then find the value of each of a and b

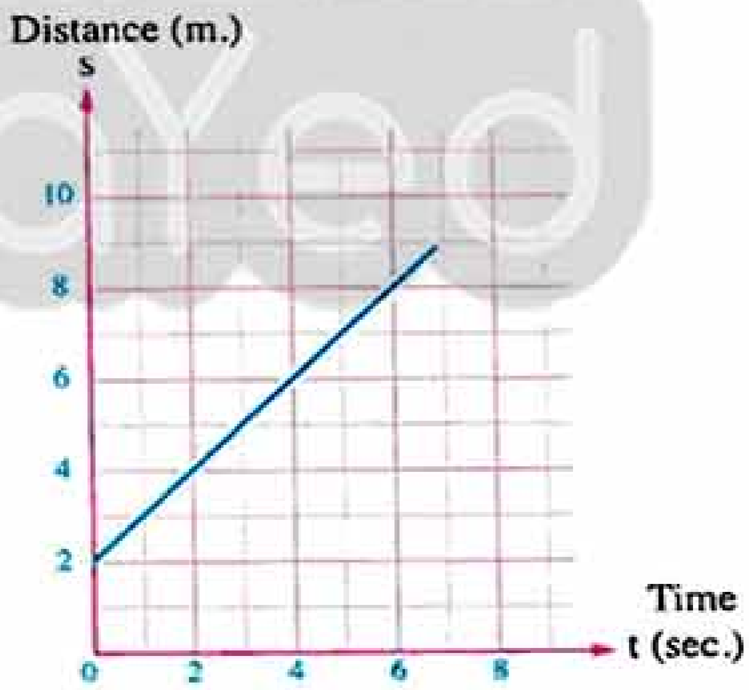
4

[a] If the slope of the straight line which passes through the two points (3 , a) and (5 , 4) equals 3 , find the value of a

[b] Prove that the points A (2 , – 3) , B (4 , – 5) and C (0 , – 1) are collinear.

5

The opposite graph represents the relation between the distance (s) in metres which a particle away from the observer and the elapsed time(t) in seconds.



First : Find the distance between the particle and the observer :

1 at beginning the motion.

2 after t = 6 sec.

Second : Find the slope of the straight line which represents the relation.

Summary of Unit 3

- You can represent the frequency table with sets by the ascending or the descending cumulative frequency curves.
- The range is the difference between the greatest value and the smallest value.
- The mean of a set of values $= \dfrac{\text{The sum of values}}{\text{Number of values}}$
- The mean of frequency distribution with sets $= \dfrac{\text{The sum of } (X \times f)}{\text{The sum of } f}$

  where f is frequency and X is the centre of the set and equals $\dfrac{\text{its lower limit + its upper limit}}{2}$
- The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it ,

  if the values number is odd , then the median is the value lying in the middle exactly ,
  if the values number is even , then the median $= \dfrac{\text{The sum of the two values lying in the middle}}{2}$
- The intersection point of the ascending and the descending cumulative frequency curves determines the median on the sets axis.
- The mode of a set of values is the most common value in the set , or it is the value which is repeated more than any other values.

Exams on Unit Three

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. The median of the values : 15 , 22 , 9 , 11 , 33 is ....

   (a) 9 (b) 15 (c) 18 (d) 90

2. The arithmetic mean of the values : 19 , 32 , 27 , 6 , 6 is ....

   (a) 90 (b) 32 (c) 18 (d) 6

3. It the mode of the values : 4 , 5 , a , 3 is 3 , then a = ...

   (a) 3 (b) 4 (c) 5 (d) 6

4. If the median of the values : k + 1 , k + 2 , k + 5 , k + 4 , k + 3 is 13 , then k = ...

   (a) 2 (b) 5 (c) 10 (d) 13

5. If the arithmetic mean of the marks of five pupils is 30 , then the sum of their marks equals .... marks.

   (a) 15 (b) 6 (c) 100 (d) 150

2 Complete the following :

1. If the order of the median of a set of values is the fifth , then the number of values equals ....
2. If the mode of the values : 15 , 9 , x + 6 , 9 , 15 is 9 , then x = ...
3. The point of intersection of the ascending and descending cumulative curves determines .... on the horizontal axis.
4. If the arithmetic mean of the values : 1 , 6 , 4 , 4 , 5 k is 7 , then k = ...
5. The centre of the set whose lower boundary is 2 and its upper boundary is 6 , is ....

3 **The following table shows the frequency distribution of marks of 10 students in a methematics exam :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	1	2	4	2	1	10

1 Find the arithmetic mean of marks.

2 If the mark of success is 30 , find the number of failure students.

4 **Find using the following frequency distribution :**

Sets	0 –	2 –	4 –	6 –	k –	Total
Frequency	m	5	8	7	6	30

1 The values of k and m.

2 The median using the ascending cumulative frequency curve.

5 **Find the mode of the following frequency distribution of marks of 40 students in an exam :**

Sets of marks	30 –	40 –	50 –	60 –	70 –	80 –	Total
Frequency	4	8	12	7	5	4	40

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The order of the median of the values : 4 , 5 , 6 , 7 and 8 is the ....

(a) third. (b) fourth. (c) fifth. (d) sixth.

2 If the arithmetic mean of the values : 18 , 23 , 29 , 2 k – 1 and k is 18 , then k = ...

(a) 1 (b) 7 (c) 29 (d) 90

3 The mode of the values : 14 , 11 , 10 , 11 , 14 , 15 and 11 is ....

(a) 14 (b) 10 (c) 11 (d) 15

4 The arithmetic mean of the values : 3 – a , 5 , 1 , 4 and 2 + a equals ....

(a) 1 (b) 2 (c) 3 (d) 15

5 If the centre of a set is 10 and its lower boundary is 4 , then its upper boundary is ....

(a) 10 (b) 4 (c) 7 (d) 16

Unit 3

2 Complete the following :

1. The point of intersection of the ascending and the descending cummulative frequency curves determines .... on the vertical axis.
2. The most common value of a set of values is called ....
3. If the arithmetic mean of a frequency distribution is 35.7 and the total of frequencies is 200 , then the total of the products of frequencies of each set by its centre is ....
4. If the order of the median of a set of values is the ninth , then the number of these values is ....
5. If the mode of the values : 13 , 7 , $x + 2$, 7 and 13 is 7 , then x = ...

3 Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	12	10	5	40

4 Find using the following frequency distribution :

Sets	0 –	2 –	k –	6 –	8 –	Total
Frequency	3	4	7	m + 2	1	20

1. The values of k and m
2. The median using the descending cummulative frequency curve of this distribution.

5 600 workers at a factory , a sample of 120 workers is chosen such that it represents the society completely to be found that their ages are distributed as the following table :

Sets of ages	25 –	30 –	35 –	40 –	45 –	50 –	Total
Number of workers	12	16	18	40	25	9	120

Graph the histogram , then find the mode age.

Quiz 1 on lesson 1 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1. If $\sqrt[3]{x} = \frac{1}{4}$, then $x =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{16}$ (c) $\frac{1}{12}$ (d) $\frac{1}{64}$

2. The diameter length of the sphere whose volume $= 36\,\pi$ cm^3 is .... cm.

(The volume of the sphere $= \frac{4}{3}\pi r^3$)

(a) 3 (b) 6 (c) 9 (d) 27

3. $\sqrt[3]{64} = \sqrt{\text{..............}}$

(a) 4 (b) 8 (c) 16 (d) 36

2 [a] Find the solution set of each of the following equations in $\mathbb{Q}$:

1. $2x^3 - 1 = 53$
2. $(5x - 3)^3 = 8$

[b] A cubic vessel has a capacity 8 litres. Calculate the length of its inner edge in cm.

Quiz 2 till lesson 2 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1. The irrational number between 3 and 4 is ....

(a) 3.6 (b) $\sqrt{6}$ (c) $\sqrt{15}$ (d) $\sqrt{17}$

2. The square whose side length is $\sqrt{7}$ cm. , its area = ... cm^2

(a) 28 (b) 49 (c) 7 (d) 14

3. If $x \in \mathbb{Z}$ and $x < \sqrt{11} < x + 1$, then $x =$ ....

(a) 3 (b) 2 (c) 4 (d) 10

2 [a] Prove that : $\sqrt{5}$ is included between 2.2 and 2.3

[b] Determine the point which represents the number $1 + \sqrt{3}$ on the number line.

Quiz 3 till lesson 3 – unit 1

time 15 min.

1 Complete the following :

1 The solution set in $\mathbb{R}$ of the equation : $x^2 + 9 = 0$ is ....

2 $\mathbb{R} - \mathbb{Q} =$ ....

3 If $\sqrt[3]{x} = -5$, then $x =$ ....

2 [a] Solve each of the following equations to the nearest one decimal given that $x \in \mathbb{R}$:

1 $\frac{1}{2} x^2 - 3 = \text{zero}$

2 $(x-3)^3 = 5$

[b] Write three irrational positive numbers less than 3

Quiz 4 till lesson 4 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 $[-3 , 2] - \{-3 , 6\} =$ ....

(a) $]-3 , 6[$ (b) $]-3 , 2[$ (c) $]-3 , 2]$ (d) $\varnothing$

2 If $\sqrt{7} \in]x , x+1[$ where $x \in \mathbb{Z}$, then $x =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

3 $\mathbb{R}_+ \cap [-1 , 3] =$ ....

(a) $]0 , 3[$ (b) $]0 , 3]$ (c) $[0 , 3]$ (d) $[0 , 3[$

2 [a] If $X =]-\infty , 1[$ and $Y = [-2 , 4[$, find each of the following as an interval using the number line :

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

4 $\grave{X}$

[b] Find the solution set of the following equation in $\mathbb{R}$:

$3x^2 + 125 = 221$

Quiz 5 till lesson 5 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 If $x^3 + 9 = 1$ where $x \in \mathbb{R}$, then $x =$ ....

(a) -8 (b) -2 (c) 2 (d) 8

2 If $x = \sqrt{3} + 2$, then $x^2 =$ ....

(a) 5 (b) 7 (c) $7 + 2\sqrt{3}$ (d) $7 + 4\sqrt{3}$

3 If $x^2 - y^2 = 60$, $x + y = 5\sqrt{6}$, then $= x - y =$ ....

(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$

2 [a] If $x = -\sqrt{3}$ and $y = 2\sqrt{3} - 3$, find the value of each of :

1 $x + y$ 2 $\frac{y}{x}$

[b] If $X = [-1 , 5[$, $Y =]1 , 7[$

, find each of the following as an interval using the number line :

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

Quiz 6 till lesson 6 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 $(\sqrt{8} + \sqrt{2})^2 =$ ....

(a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$

2 $\frac{1}{2}\sqrt{20} - \sqrt{5} =$ ....

(a) $\frac{1}{2}\sqrt{15}$ (b) $\sqrt{5}$ (c) zero (d) 1

3 If π is the ratio between the circumference of the circle and its diameter length ,
then $\pi \in$ ....

(a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{Q}$ (d) $\mathbb{Q}'$

2 [a] Put in the simplest form : $\sqrt{75} - 2\sqrt{27} + 3\sqrt{\frac{1}{3}}$

[b] Find the solution set of the following equation in $\mathbb{R}$: $\frac{1}{6}x^2 + 6 = 6\frac{1}{2}$

Quiz 7 till lesson 7 – unit 1

time 15 min.

1 Complete the following :

1 The multiplicative inverse of the number $\left(\sqrt{3}+\sqrt{2}\right)$ in the simplest form is ....

2 The rectangle whose dimensions are $\left(\sqrt{3}+1\right)$ cm. and $\left(\sqrt{3}-1\right)$ cm. has an area = ... cm^2.

3 $[1 , 5] - \{1 , 5\} =$ ....

2 [a] If $x=\sqrt{5}+\sqrt{3}$ and $y=\frac{2}{\sqrt{5}+\sqrt{3}}$,

find the value of the expression : $x^2 - 2xy + y^2$

[b] Find in the shape of an interval using the number line : $]-\infty , 1[\cup]2 , \infty[$

Quiz 8 till lesson 8 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 If $X =]-\infty , 0[$, then $\grave{X} =$ ....

(a) $\mathbb{R}_+$ (b) $[0 , \infty[$ (c) $]-\infty , 0]$ (d) $\mathbb{R}_-$

2 $\frac{\sqrt[3]{24}}{\sqrt[3]{3}} =$ ....

(a) $2\sqrt[3]{2}$ (b) 8 (c) 3 (d) 2

3 If $x=\sqrt{3}+\sqrt{2}$ and $xy = 1$, then $y =$ ....

(a) $\sqrt{2}-\sqrt{3}$ (b) $\sqrt{3}+\sqrt{2}$ (c) $\sqrt{3}-\sqrt{2}$ (d) 1

2 [a] If $x = 2+\sqrt[3]{4}$ and $y = 2-\sqrt[3]{4}$, find the value of : $\left(\frac{x-y}{x+y}\right)^3$

[b] Put in the simplest form : $\sqrt{18}-\sqrt[3]{72}-\sqrt{8}+\sqrt[3]{9}$

Quiz 9 till lesson 9 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 The circle whose radius length $=\sqrt{14}$ cm. has an area = ... cm^2.

(a) 14π (b) $2\sqrt{14}\pi$ (c) 14 (d) $2\sqrt{14}$

2 The right circular cylinder whose base radius length = 3 cm. and its height = 5 cm. its volume = ... cm^3.

(a) 15π (b) 75π (c) 45π (d) $\frac{3}{5}\pi$

3 The conjugate of the number $\frac{2}{\sqrt{5}+\sqrt{3}}$ = ...

(a) $\frac{\sqrt{5}+\sqrt{3}}{2}$ (b) $\sqrt{5}-\sqrt{3}$ (c) $\sqrt{5}+\sqrt{3}$ (d) $\frac{\sqrt{5}-\sqrt{3}}{2}$

2 [a] The height of a right circular cylinder equals its radius length , and its volume $=27\pi\ cm^3$. Calculate the lateral area of the cylinder.

[b] If $x=\sqrt{5}-\sqrt{3}$, $y=\frac{2}{\sqrt{5}-\sqrt{3}}$, **then find the value of :** $x^2+2xy+y^2$

Quiz 10 till lesson 10 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 The S.S. in $\mathbb{R}$ of the inequality $-1<-x\leq 1$ is ....

(a) $]-1 , 1]$ (b) $[-1 , 1]$ (c) $[-1 , 1[$ (d) $]-1 , 1[$

2 If three quarters of the volume of a sphere is $8\pi\ cm^3$, then the length of its radius equals .... cm.

(a) 64 (b) 8 (c) 4 (d) 2

3 $\sqrt[3]{2}+\sqrt[3]{2}=$ ....

(a) $\sqrt[3]{4}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{16}$ (d) $\sqrt[3]{2}$

2 [a] **Find in $\mathbb{R}$ the S.S. of the following inequality and represent it on the number line :**

$-1<1-2x\leq 5$

[b] **Simplify to the simplest form :** $2\sqrt{5}+9\sqrt{\frac{1}{3}}-\sqrt{27}-5\sqrt{\frac{1}{5}}$

Quiz 11 till lesson 1 – unit 2

time 15 min.

1 Choose the correct answer from the given ones :

1. The ordered pair that satisfies the relation : $3x - y = 1$ is ....

(a) (0 , 5) (b) (– 1 , 2) (c) (1 , 2) (d) (2 , 1)

2. If (2 k , 3 k) satisfies the relation : $x + y = 15$, then k = ...

(a) 5 (b) 3 (c) – 5 (d) – 3

3. If the ordered pair (1 , 3) satisfies the relation : $y = 3x + c$, then c = ...

(a) zero (b) 1 (c) 2 (d) 3

2 [a] Find three ordered pairs satisfying the relation :

$2x - 3y = 6$, then represent it graphically.

[b] If the straight line : $y - 3x = a$ intersects the x-axis at the point (1 , b) , then find the value of each of : a and b

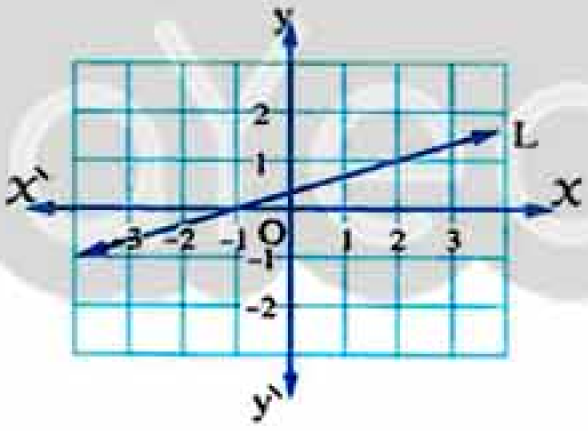
Quiz 12 till lesson 2 – unit 2

time 15 min.

1 Complete the following :

1. If $-\sqrt{25} = \sqrt[3]{x}$, then x = ...

2. The slope of the straight line L in the opposite graph is ....



3. The slope of any straight line parallel to y-axis is ....

2 [a] Represent the straight line that represents the relation : $2x + y = 4$, if this line intersects the x-axis at the point A and intersects the y-axis at the point B , then find the area of Δ AOB , where O is the origin point

[b] Prove that :

The points A , B and C are collinear where A (1 , 1) , B (– 5 , – 11) and C (4 , 7)

Quiz 13 till lesson 3 – unit 2

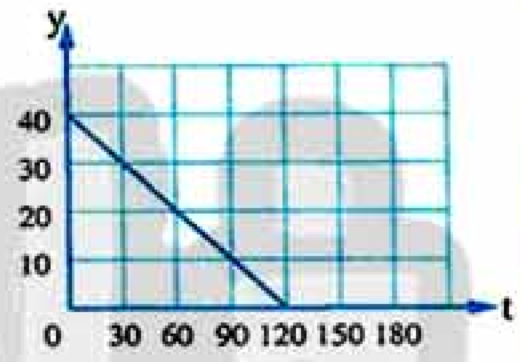
time 15 min.

1 Choose the correct answer from the given ones :

1. If the ordered pair (5 , 2) satisfies the relation $y = 2x - b$, then b = ...
   (a) –1 (b) 3 (c) 8 (d) 4
2. The slope of the straight line passes through the two points (3 , 4) and (3 , – 5) is ....
   (a) zero (b) unknown (c) $\frac{9}{6}$ (d) $\frac{2}{3}$
3. The perimeter of a face of a cube is 12 cm. Its lateral area = ... cm^2.
   (a) 144 (b) 64 (c) 36 (d) 54

2 [a] A right cylinder whose volume is 3080 cm^3 , and its height is 20 cm. , find the diameter length of its base. $\left(\pi = \frac{22}{7}\right)$

[b] Ahmed filled the tank of his car by fuel the opposite graph represent the relation between the time (t) in minutes and the amount of remained fuel in the tank (y) in litre , from the graph :



1. What is the greatest capacity of the tank ?
2. What is the average of the fuel consumption per minutes ?
3. When the tank get empty ?

Quiz 14 till lesson 1 – unit 3

time 15 min.

1 Choose the correct answer from the given ones :

1. The S.S. of the equation : $\sqrt{2}\,x - 1 = 3$ in $\mathbb{R}$ is ....
   (a) $\{2\}$ (b) $\{\sqrt{2}\}$ (c) $\{2\sqrt{2}\}$ (d) $\{4\sqrt{2}\}$
2. If $x = \sqrt{3} + 2$, $y = \sqrt{3} - 2$, then $(xy , x + y)$ = ...
   (a) $(1 , 2\sqrt{3})$ (b) $(-1 , 2\sqrt{3})$ (c) $(5 , 2\sqrt{3})$ (d) (5 , 9)
3. If (– 1 , 5) satisfies the relation : $3x + ky = 7$, then k = ...
   (a) 2 (b) – 2 (c) 1 (d) 10

2 The following table shows the marks obtained by 30 students in an examination :

5	9	11	4	9	9	16	7	8	12	2	10	7	12	5
8	15	13	13	9	7	14	19	3	11	14	3	12	13	17

Form the frequency table to these data.

Quiz 15 till lesson 2 – unit 3

time 15 min.

1 Choose the correct answer from the given ones :

1 If $x^3 + 9 = 1$, where $x \in \mathbb{R}$, then $x =$ ……………

(a) -8 (b) -2 (c) 2 (d) 8

2 The slope of the straight line passes through A (2 , 3) and B (0 , 1) is ……………

(a) 2 (b) -2 (c) 1 (d) -1

3 $\sqrt[3]{54} + \sqrt[3]{-2} =$ ……………

(a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$

2 The following table shows the frequency distribution of wages of 100 workers weekly :

Sets	50 –	60 –	70 –	80 –	90 –	Total
Frequency	5	15	30	40	10	100

1 Find the number of workers whose wages are less than 70 pounds weekly.

2 Graph the ascending cummulative frequency curve.

Quiz 16 till lesson 3 – unit 3

time 15 min.

1 Complete the following :

1 If the mean of the values : 27 , 8 , 16 , 24 , 6 and k is 14 , then k = ……………

2 The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is ……………

3 If the arithmetic mean of a frequency distribution is 12.3 and the sum of all frequences is 100 , then the sum of the products of each frequence and the centre of its set = ……………

2 [a] The following table shows the frequency distribution of extra wages weekly for 100 workers in a factory :

Extra wages in pounds	20 –	30 –	40 –	50 –	60 –	70 –
Number of workers	10	14	k	k + 4	20	8

1 Calculate the value of k

2 Find the arithmetic mean of this distribution.

[b] If $x = \sqrt{7} + \sqrt{5}$ and $y = \frac{2}{x}$, find the value of the expression : $\frac{x + y}{x\,y}$ in its simplest form.

Revision for the important rules of algebra and statistics

First Real numbers

Remember that

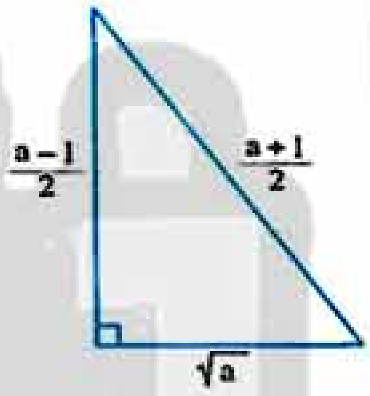
- $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$
- $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$
- $\mathbb{R} - \mathbb{Q} = \grave{\mathbb{Q}}$
- $\mathbb{R} - \grave{\mathbb{Q}} = \mathbb{Q}$
- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- $\pi \in \grave{\mathbb{Q}}$
- $\mathbb{R}^* = \mathbb{R} - \{0\}$

Remember The representing of the irrational number on the number line

Each irrational number can be represented by a point on the number line.

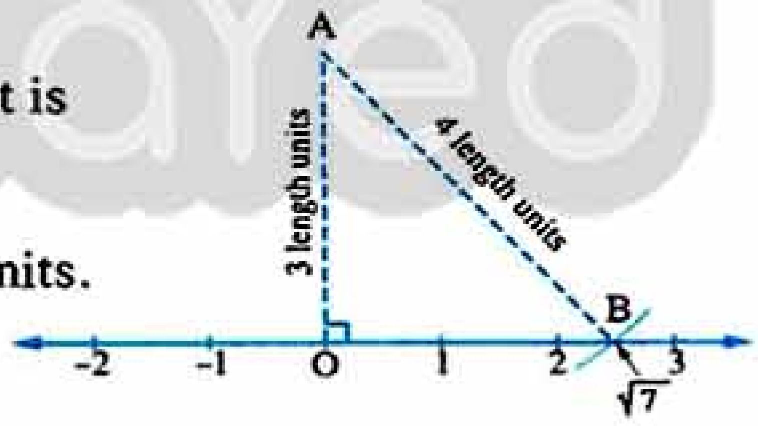
and to draw a line segment with length $= \sqrt{a}$ length unit where $a > 1$

Draw a right-angled triangle in which :



- The length of one side of the right-angle $= \frac{a-1}{2}$ length unit.
- The length of the hypotenuse $= \frac{a+1}{2}$ length unit.

and we can apply this to represent the irrational number $\sqrt{7}$ on the number line as the following :



- From the point which represents the number zero on the number line , we draw a perpendicular line segment and it is $\overline{OA}$ where $OA = \frac{7-1}{2} = 3$ length units.
- Using the compasses with a distance $= \frac{7+1}{2} = 4$ length units. and centre at A then draw an arc to cut the number line on the right side of the point O at the point B , then B is the point which represents $\sqrt{7}$ as in the figure.
- Notice that : To represent the number $\left(-\sqrt{7}\right)$, we draw the arc which cuts the number line on its left side , not on its right side.
- Notice that : To represent the number $\left(1+\sqrt{7}\right)$, we follow the same previous steps but we draw the perpendicular line segment $\overline{OA}$ from the point which represents the number 1 , not the number 0

Remember The operations on intervals

Intervals	Intersection	Union	Difference	Complement
$X = [-1, 5[$ $, Y =]-3, 2[$	$X \cap Y = [-1, 2[$	$X \cup Y =]-3, 5[$	$X - Y = [2, 5[$ $, Y - X =]-3, -1[$	$\grave{X} = \mathbb{R} - [-1, 5[$ $=]-\infty, -1[\cup [5, \infty[$
$X =]-\infty, 1]$ $, Y = [-2, 1[$	$X \cap Y = [-2, 1[$	$X \cup Y =]-\infty, 1]$	$X - Y$ $=]-\infty, -2[\cup \{1\}$ $, Y - X = \varnothing$	$\grave{X} =]1, \infty[$
$X = [-1, 5]$ $, Y =]-1, 5[$	$X \cap Y =]-1, 5[$	$X \cup Y = [-1, 5]$	$X - Y = \{-1, 5\}$ $, Y - X = \varnothing$	$\grave{Y} = \mathbb{R} -]-1, 5[$ $=]-\infty, -1] \cup [5, \infty[$
$X =]-3, 4]$ $, Y = \{-3, 4\}$	$X \cap Y = \{4\}$	$X \cup Y = [-3, 4]$	$X - Y =]-3, 4[$ $, Y - X = \{-3\}$	$\grave{Y} = \mathbb{R} - \{-3, 4\}$

Remember The operations on the square roots and the cube roots

The square roots

① $\sqrt{a}\times\sqrt{b}=\sqrt{a\,b}$ *For Example :* $\sqrt{3}\times\sqrt{12}=\sqrt{3\times 12}=\sqrt{36}=6$

② $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$ (where $b\neq 0$) *For Example :* $\frac{\sqrt{8}}{\sqrt{2}}=\sqrt{\frac{8}{2}}=\sqrt{4}=2$

③ $\frac{\sqrt{a}}{\sqrt{b}}=\frac{\sqrt{a}}{\sqrt{b}}\times\frac{\sqrt{b}}{\sqrt{b}}=\frac{\sqrt{a\,b}}{b}$ (where $b\neq 0$) *For Example :* $\frac{\sqrt{2}}{\sqrt{5}}=\frac{\sqrt{2}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{10}}{5}$

The cube roots

① $\sqrt[3]{a}\times\sqrt[3]{b}=\sqrt[3]{a\,b}$ *For Example :* $\sqrt[3]{3}\times\sqrt[3]{9}=\sqrt[3]{3\times 9}=\sqrt[3]{27}=3$

② $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}=\sqrt[3]{\frac{a}{b}}$ (where $b\neq 0$) *For Example :* $\frac{\sqrt[3]{32}}{\sqrt[3]{4}}=\sqrt[3]{\frac{32}{4}}=\sqrt[3]{8}=2$

Example Simplify to the simplest form :

1. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}$
2. $\sqrt{18}-\frac{\sqrt{12}}{\sqrt{6}}$
3. $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)$
4. $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}$
5. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}$

Solution

1. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}=\sqrt{2\times 16}-\sqrt{2\times 36}+3\times 2\sqrt{\frac{1}{2}}$
$=4\sqrt{2}-6\sqrt{2}+3\sqrt{\frac{1}{2}\times 4}=4\sqrt{2}-6\sqrt{2}+3\sqrt{2}=\sqrt{2}$

2. $\sqrt{18}-\frac{\sqrt{12}}{\sqrt{6}}=\sqrt{2\times 9}-\sqrt{2}=3\sqrt{2}-\sqrt{2}=2\sqrt{2}$

3. $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)=5\sqrt{2}\times 2\sqrt{2}+5\sqrt{2}\times\sqrt{12}=10\sqrt{4}+5\sqrt{24}=10\times 2+5\sqrt{4\times 6}$
$=20+5\times 2\sqrt{6}=20+10\sqrt{6}$

4. $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=\sqrt[3]{2\times 27}+6\sqrt[3]{8\times 2}-3\times 2\sqrt[3]{\frac{1}{4}}$
$=3\sqrt[3]{2}+6\times 2\times\sqrt[3]{2}-3\times\sqrt[3]{8\times\frac{1}{4}}=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

5. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}=\sqrt[3]{8\times 9}+\sqrt[3]{\frac{1}{3}\times\frac{9}{9}}-\sqrt[3]{9}$
$=2\sqrt[3]{9}+\sqrt[3]{\frac{9}{27}}-\sqrt[3]{9}=2\sqrt[3]{9}+\frac{1}{3}\sqrt[3]{9}-\sqrt[3]{9}=\frac{4}{3}\sqrt[3]{9}$

Remember — The two conjugate numbers

If a and b are two positive rational numbers :
then each of the two numbers $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$
is conjugate to the other one and we find that :

- Their sum $=\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)=2\sqrt{a}=$ twice the first term
- Their product $=\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2=a-b$
  = The square of the first term – the square of the second term

For example : The number $\left(\sqrt{3}-\sqrt{2}\right)$ its conjugate is $\left(\sqrt{3}+\sqrt{2}\right)$, then we find that :

- Their sum $=2\sqrt{3}$
- Their product $=3-2=1$

Remark

If we have a real number whose denominator is written in the form $\left(\sqrt{a}+\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

For example :

For writting the number $\frac{12}{\sqrt{6}-\sqrt{2}}$ in the simplest form , we multiply the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{6}+\sqrt{2}\right)$

$$\therefore \frac{12}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}=\frac{12\left(\sqrt{6}+\sqrt{2}\right)}{6-2}=3\left(\sqrt{6}+\sqrt{2}\right)=3\sqrt{6}+3\sqrt{2}$$

Important remarks from multiplying by inspection

- **We know that :** $(x-y)(x+y)=x^2-y^2$
- **And we know also :**

$(x+y)^2=x^2+2xy+y^2$

Then

- $x^2+xy+y^2=(x+y)^2-xy$
- $x^2+y^2=(x+y)^2-2xy$

or

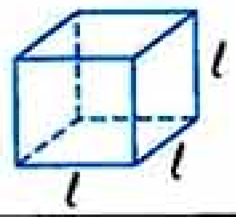
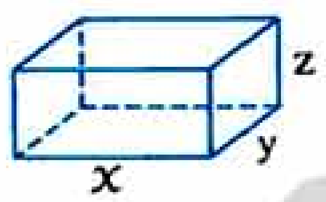
$(x-y)^2=x^2-2xy+y^2$

Then

- $x^2-xy+y^2=(x-y)^2+xy$

$x^2+y^2=(x-y)^2+2xy$

Summary of rules of areas and volumes of some solids

The solid		The lateral area	Total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh+2\pi r^2$ $=2\pi r(h+r)$	$\pi r^2 h$
The sphere			$4\pi r^2$	$\frac{4}{3}\pi r^3$

Remember that : The circumference of the circle $= 2\pi r$, the area of the circle $= \pi r^2$

Remember — Solving equation of the first degree in one unknown in $\mathbb{R}$

- Solving the equation of the first degree in one unknown in $\mathbb{R}$ means finding the real number which satisfies this equation.

And the following example shows how to solve an equation of the first degree in one unknown.

Example

Find in $\mathbb{R}$ the solution set of each of the following equations , then represent the solution on the number line :

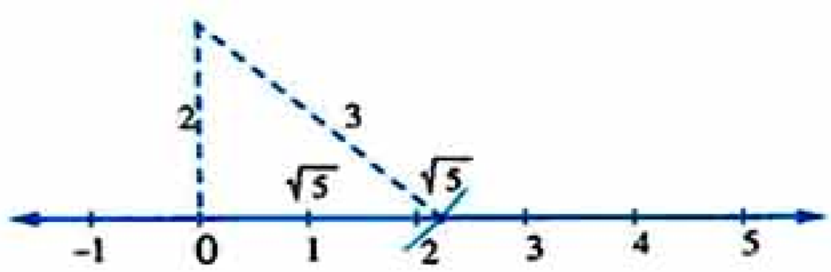
1. $\sqrt{5}\,x - 1 = 4$
2. $x - \sqrt{3} = 2$

Solution

1. $\because \sqrt{5}\,x - 1 = 4 \quad \therefore \sqrt{5}\,x = 4 + 1 = 5$

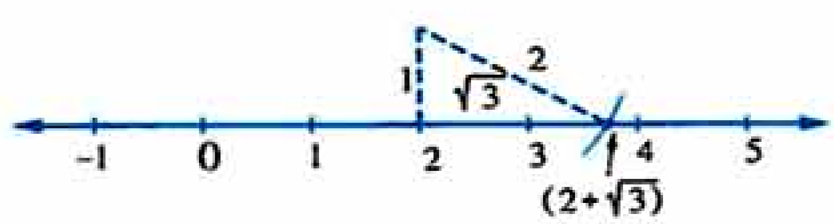
$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

$\therefore$ The S.S. $= \{\sqrt{5}\}$



2. $\because x - \sqrt{3} = 2 \quad \therefore x = 2 + \sqrt{3}$

$\therefore$ The S.S. $= \{2 + \sqrt{3}\}$



Remember **Solving inequality of the first degree in one unknown in $\mathbb{R}$**

- Solving the inequality means finding all values of the unknown which satisfy this inequality.
- The solution set of the inequality in $\mathbb{R}$ will be written as an interval

And the following example shows how to solve an inequality of the first degree in one unknown in $\mathbb{R}$

Example

Find in $\mathbb{R}$ the solution set of each of the following inequalities , then represent the solution on the number line :

1. $2x + 6 < 2$
2. $5 - 4x \le -3$
3. $3 < 3 - 5x < 13$
4. $x - 2 \ge 3x - 5$

Solution

1 $\because 2x + 6 < 2$ $\therefore 2x < 2 - 6$

$\therefore 2x < -4$ $\therefore x < \frac{-4}{2}$

$\therefore x < -2$ $\therefore$ The S.S. $=]-\infty , -2[$

2 $\because 5 - 4x \le -3$ $\therefore -4x \le -8$

$\therefore x \ge \frac{-8}{-4}$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

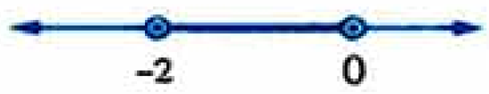
$\therefore x \ge 2$ $\therefore$ The S.S. $= [2 , \infty[$

3 $\because 3 < 3 - 5x < 13$ (adding -3 to all sides)

$\therefore 0 < -5x < 10$ (dividing all sides by -5) $\therefore 0 > x > -2$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

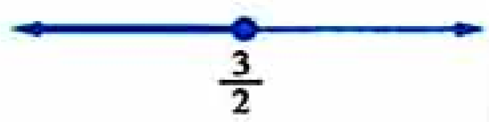
$\therefore$ The S.S. $=]-2 , 0[$



4 $\because x - 2 \ge 3x - 5$ $\therefore x - 3x \ge -5 + 2$

$\therefore -2x \ge -3$ $\therefore x \le \frac{3}{2}$

$\therefore$ The S.S. $=]-\infty , \frac{3}{2}]$



Second Relation between two variables

Remember The linear relation

It is a relation of the first degree between two variables x and y , it is in the form :
$a x + b y = c$, where a , b and c are real numbers , a and b $\neq 0$ together.
And there is an infinite number of ordered pairs which satisfy this relation , and it is enough to get three ordered pairs satisfying the relation at the graphical representation.

Example 1

Find three ordered pairs satisfying the relation : $3x - 2y = 6$

Solution

$\because 3x - 2y = 6$ $\therefore -2y = 6 - 3x$ $\therefore y = \frac{3x - 6}{2}$

- Putting $x = 0$ $\therefore y = -3$ $\therefore (0 , -3)$ satisfies the relation.
- Putting $x = 1$ $\therefore y = -\frac{3}{2}$ $\therefore \left(1 , -\frac{3}{2}\right)$ satisfies the relation.
- Putting $x = 2$ $\therefore y = 0$ $\therefore (2 , 0)$ satisfies the relation.

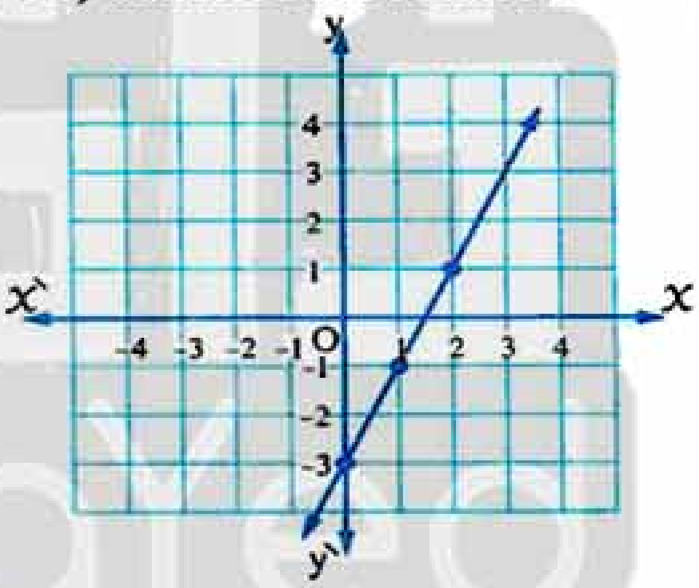
Example 2

Represent graphically the relation : $2x - y = 3$

Solution

$\because 2x - y = 3$ $\therefore y = 2x - 3$

x	0	1	2
y	-3	-1	1



Remember The slope of the straight line

The slope of the straight line = $\frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

i.e. $S = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

For example : The slope of the straight line passing through the two points : (2 , 3) , (– 5 , 2) is :

$S = \frac{2 - 3}{-5 - 2} = \frac{-1}{-7} = \frac{1}{7}$

Notice that

- The slope of the straight line parallel to x-axis = 0
- The slope of the straight line parallel to y-axis is undefined.

Third Statistics

Remember — The tables and cumulative frequency curves

The following frequency table shows the weekly wages in pounds of 50 workers in a factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (frequency)	5	12	22	7	4	50

1 Forming the ascending cumulative frequency table and graphing the curve

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (frequency)	5	12	22	7	4

Less than 54 = 0
Less than 58 = 5 + 0 = 5
Less than 62 = 5 + 12 = 17
Less than 66 = 5 + 12 + 22 = 39
Less than 70 = 5 + 12 + 22 + 7 = 46
Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

"The ascending cumulative frequency table"

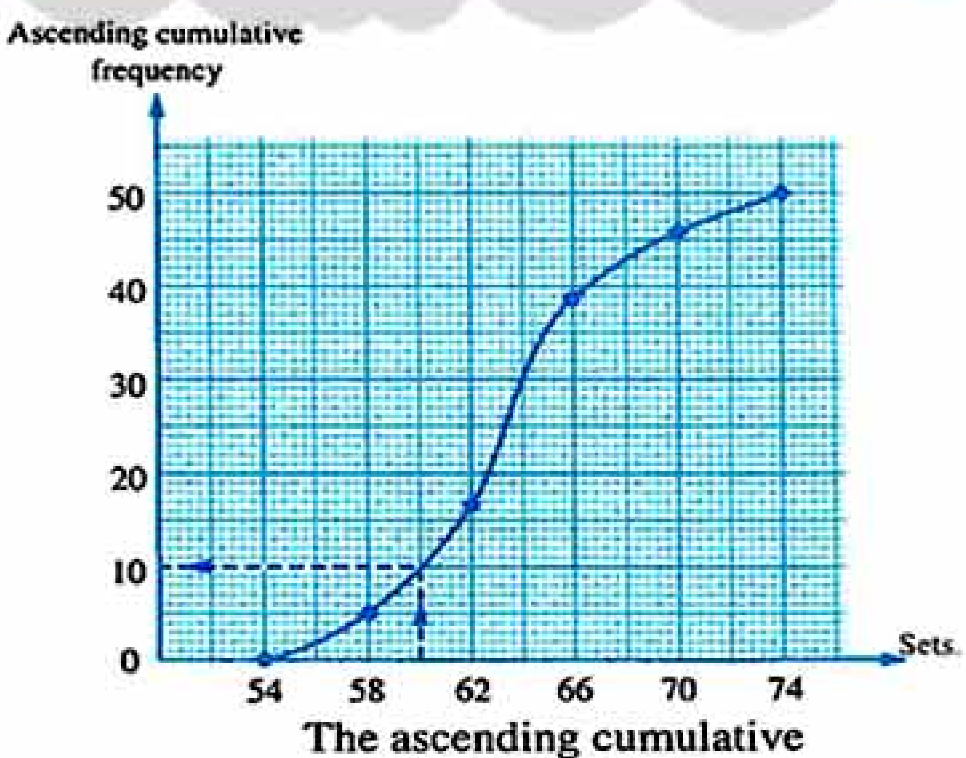
Notice that

The ascending cumulative frequency begins with zero and ends at the total frequency.

- From the opposite graph , we can find the number of individuals which less than a certain value.

For Example :

The number of workers whose wages are less than 60 pounds is 10 workers



The ascending cumulative frequency curve

2 Forming the descending cumulative frequency table and graphing the curve

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (frequency)	5	12	22	7	4

54 and more = 5 + 12 + 22 + 7 + 4 = 50

58 and more = 12 + 22 + 7 + 4 = 45

62 and more = 22 + 7 + 4 = 33

66 and more = 7 + 4 = 11

70 and more = 4

74 and more = 0

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

"The descending cumulative frequency table"

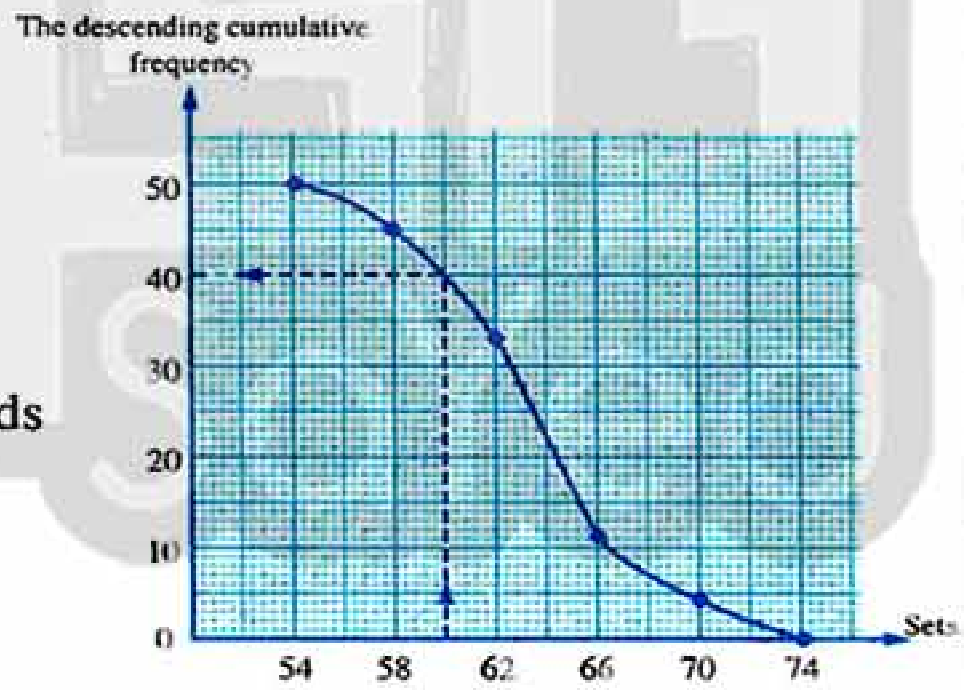
Notice that

The descending cumulative frequency begins with the total frequency and ends with zero.

- From the opposite graph , we can find the number of individuals which more than or equal to a certain value.

For example :

The number of workers whose wages are 60 pounds or more = 40 workers.



The descending cumulative frequency curve

Remember **The measures of the central tendency**

(1) The mean. (2) The median. (3) The mode.

1 The mean

[a] The mean of a set of values (simple frequency distribution)

The mean of a set of values $= \dfrac{\text{The total of values}}{\text{Number of values}}$

For example : The mean of the numbers : 5 , 3 , 7 , 9 $= \dfrac{5+3+7+9}{4} = 6$

[b] The mean of a frequency distribution with sets

Example

The following table shows the distribution of the marks of 50 pupils in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

and the required is finding the mean of these marks.

Solution

1. Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

$\therefore$ The centre of the first set $= \dfrac{10+20}{2} = 15$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre $= \dfrac{50+60}{2} = 55$

2. Form the following table :

Set	Centre of the set « x »	Frequency « f »	$x \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

The mean $= \dfrac{\text{The sum of } (x \times f)}{\text{The sum of } f} = \dfrac{1700}{50} = 34$ marks.

2 The median

[a] The median of a set of values

The median is the middle value in a set of values after arranging it ascendingly or descendingly. such that the number of values which are less than it is equal to the number of values which are greater than it.

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

If the values number is even , then

The median $= \frac{\text{The sum of the two values lying in the middle}}{2}$

For example :

If the values are

42 , 23 , 17 , 30 and 20

We arrange them ascendingly as follows

17 , 20 , 23 , 30 , 42

The median = 23

For example :

If the values are

27 , 13 , 23 , 24 , 13 , 21

We arrange them ascendingly as follows

13 , 13 , $\boxed{21 , 23}$, 24 , 27

The median $= \frac{21 + 23}{2} = 22$

[b] Finding the median of a frequency distribution with sets graphically

For finding the median of a frequency distribution with sets graphically , do the following steps :

1. Form the ascending or the descending cumulative frequency table , then draw the cumulative frequency curve of it.
2. We find the order of the median $= \frac{\text{The total of frequency}}{2}$
3. Determine the point which represents the order of the median on the vertical axis, from this point , draw a horizontal straight line to cut the curve at a point, then from this point , draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.
   The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

Example

The following table shows the frequency distribution of marks of 50 students in math exam :

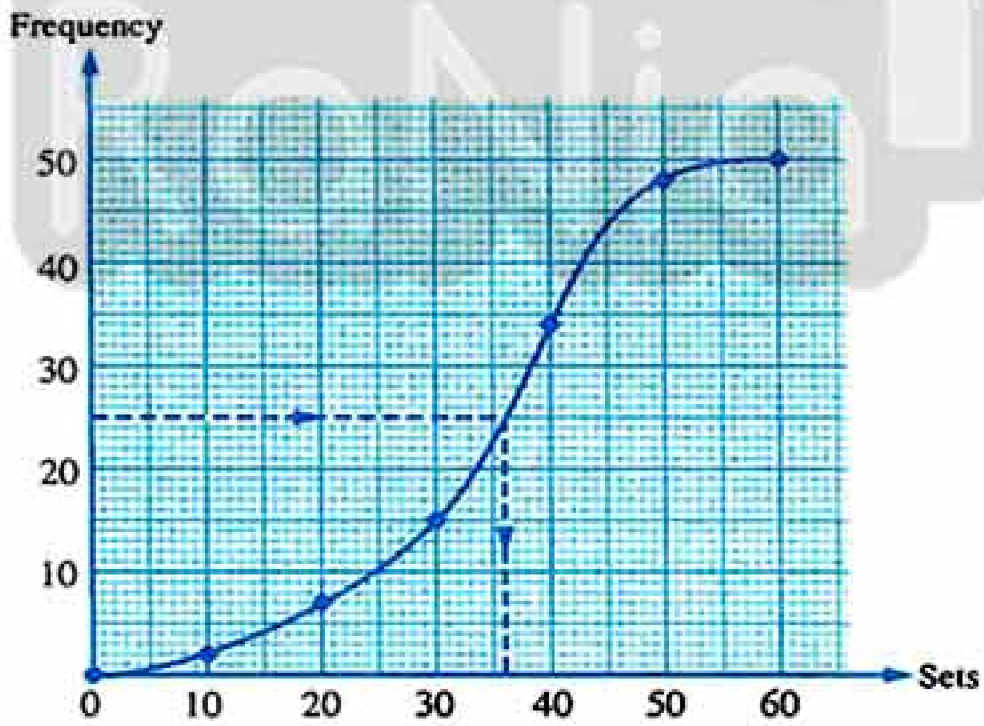
Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the student.

Solution

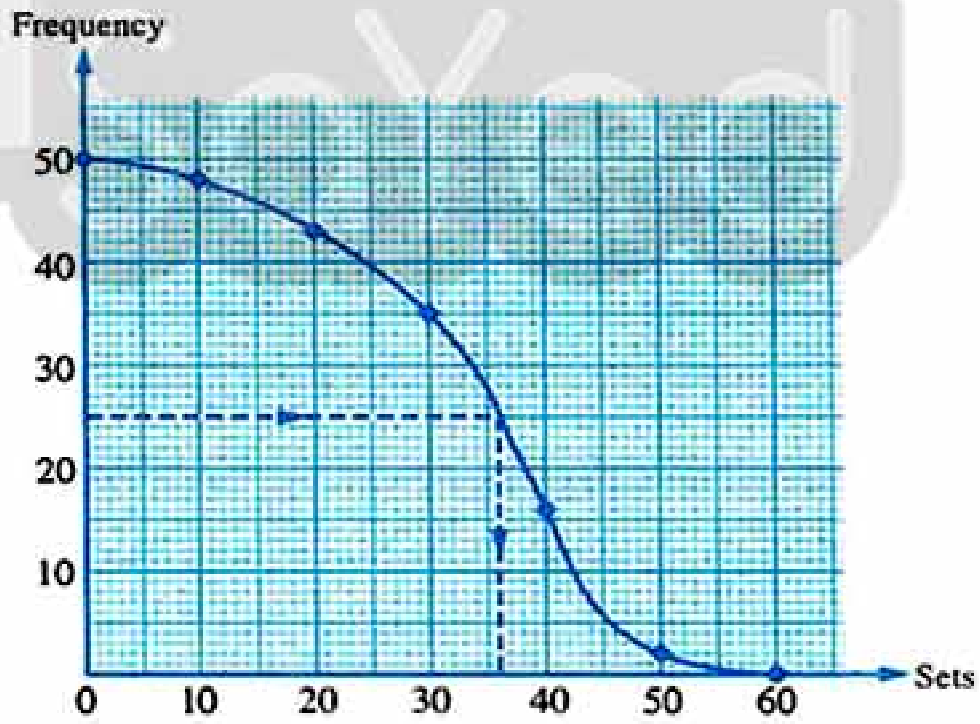
Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50



Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0



$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the two previous graphs , the median $= 36$ approximately

3 The mode

[a] The mode of a set of values

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example : The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

[b] The mode of a frequency distribution with sets

Example

The following is the frequency distribution of marks of 100 pupils in one of the exams :

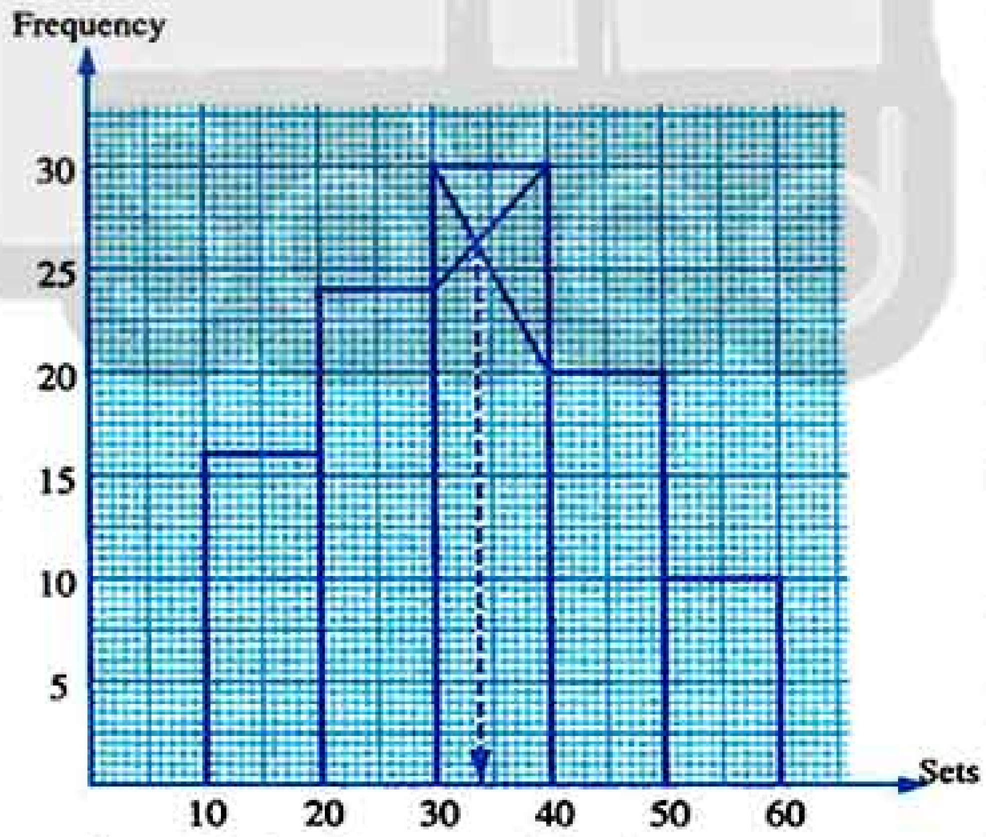
Set of marks	10 –	20 –	30 –	40 –	50 –	Total
Number of pupils	16	24	30	20	10	**100**

Find the mode mark for these pupils.

Solution

We can find the mode of that distribution graphically using the histogram as follows :

1. Draw two orthogonal axes : one of them is horizontal and the other is vertical to represent the frequency of each set.
2. Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
3. Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
4. Draw a rectangle whose base is set (10 –) and its height equals the frequency (16)
5. Draw a second rectangle adjacent to the first one whose base is set (20 –) and its height equals the frequency (24)



6. Repeat drawing the remained adjacent rectangles till the last set (50 –)
7. Determine the set which has the greatest frequency then draw two lines as shown in the histogram to intersect at a point.
   From this point , draw a vertical line to intersect the horizontal axis at a point which represents the value of the mode.
   i.e. The mode mark is 34 approximately.